Calculus II

Name: ____________________
Section: ___________________
(class meeting time)

Maple Lab #4: Improper Integrals

(Notice step 17 on the last page - the lab must be complete and correct before it can be turned in. Please feel free to get help from the lab aide at any time. If you do it someplace other than the Maple Lab at the Academic Success Center, indicate in the space provided where you worked on it.)

Up until now, when you evaluated an definite integral, there was always a finite number at the top and the bottom of the anti-derivative symbol. The lower number represented the left side of the area being evaluated, the top number the right side.

For example:

1. Evaluate: $\int_1^5 \frac{1}{x^2}dx =$

(Do by hand and show work.)

This can be done with Maple by entering the statement:

int(1/x^2, x = 1 .. 5);

2. Now suppose you wanted to find $\int_1^{10} \frac{1}{x^2}dx =$

Don't do this one by hand -- go back to your last Maple entry, and change the 5 to a 10. What is the answer this time?_______

3. Now find $\int_1^{100} \frac{1}{x^2}dx =$ ______________

plot (1/x^2, x = 0 .. 10, y = 0 .. 1.1);

4. Use this statement to see the curve under which you are finding these areas.

5. As the upper limit increases, the area under the curve increases. What do you think the area is getting closer to as the upper limit gets higher and higher?

____

How is that possible if the area keeps increasing?

____

____

____

int(1/x^2, x = 1 .. 1000);

6. Test your answer by finding these integrals:

$\int_1^{1000} \frac{1}{x^2} dx =$ _______ $\int_1^{10000} \frac{1}{x^2} dx =$ __________

Do you think you were right? __________

What we can say, is that the area under the curve y = $\frac{1}{x^2}$ as x goes from 1 to $+\infty$ is 1. We can also say that $\lim_{b \to \infty} \int_1^b \frac{1}{x^2} = 1$ or $\int_1^{\infty} \frac{1}{x^2}$ = 1. These are known as improper integrals.

plot(1/x, x = 0 .. 10, y = 0 .. 1.1);

7. Now look at the graph of a similar function f(x) = $\frac{1}{x}$.

Enlarge the graph, change it to a horizontal orientation, then print it out.

How do you think the area under this curve will compare to the area under y = $\frac{1}{x^2}$? _____________

8. To find the area under this graph from x = 1 to x = 5 evaluate: $\int_1^5 \frac{1}{x} dx =$ _________________ (first do it by hand).

This can be done with Maple by entering the statements:

```
int(1/x, x = 1 .. 5);
evalf(%);
```

9. Now suppose you wanted to find $\int_1^{10} \frac{1}{x} dx =$

Do it by hand first --then go back to your last Maple entry, and change the 5 to a 10. What is the answer this time?_______

10. Now find $\int_1^{100} \frac{1}{x} dx =$ ______________ As the upper limit gets larger, the integral seems to be getting closer to

________.

11. Let's make the upper limit even higher. Find $\int_1^{1000} \frac{1}{x} dx$

Now find $\int_1^{10000} \frac{1}{x} dx =$_______________ Are these giving you a better idea about what the area under the curve is going to as the upper limit gets higher and higher? ________ It turns out that there is no limit to this area. By taking the upper limit high enough, the area can go to any value.

In symbols, $\int_1^{\infty} \frac{1}{x} dx = +\infty$. Or we say that this integral **diverges,**

whereas $\int_1^{\infty} \frac{1}{x^2} dx$ **converges** to1.

12. Consider the integral $\int_1^{\infty} \frac{1}{x^{1.2}} dx$. Use a method similar to the above steps to show that this is a convergent integral. Explain what you did, and find the value to which the integral converges.

13. Make a generalization about the improper integral

$$\int_1^{\infty} \frac{1}{x^p}dx$$ (ie. for what values of p does this improper integral converge? For what values of p does it diverge?)

14. This lab is complete and correct.

__________________________________ ___________________
lab aide date

15. I want your opinion.

	Strongly agree				strongly disagree
The labs have helped me learn the material for the course.	5	4	3	2	1

comments:

How long did it take you to complete this lab? ____

Other comments: