

Revised 8/4/15  
Calculus I

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### Maple Lab #5: Areas as Limits

The main purpose of this Maple lab is to introduce you with the technique of finding an approximation of an area under a curve. In the end, the connection is made between the area under a curve, and the definite integral. *(Notice step 25a and 25b on the last page . When you do the lab at the Learning and Advising Center and get the signature, you are guaranteed credit for it. If you are not doing the lab at the Math Computer Lab at the Academic Success Center, indicate where you did it in the indicated space. You will be contacted if the lab is not complete and correct.)*

#### Type:

#### Directions and explanation:

`h := proc(x) 9 - x^2 end`

1. Let's work with the function  $h(x) = 9 - x^2$ .

When a function is defined in this manner (I would read it as "h of x is equal to nine minus x squared."), we can just enter `h(5)`; instead of `subs(x=5, h)`; to plug in a 5 for x.

`h(5)`

`plot(h(x), x = 0 .. 3, y = -2 .. 10)`

2. Read this entry as "Graph  $h(x)$ . Make the x axis go from zero to three, and make the y axis go from negative two to ten."

`with(student);`

3.. The "with(student)" command makes some additional commands available to you. Now, suppose you want to estimate the area under this curve from where x is 0 to where x is 3.

`rightbox(h(x),x=0..3,3)`

4. Read this entry as "*Graph  $h(x)$ . Make the x axis go from zero to three. Then draw three inscribed rectangles under the curve.*"

5. Either print the graph out, or draw it on a separate sheet of paper. (Including the boxes. ) **(Must do this for credit!)**

6. Look at the first box. Its width is 1. Its left side is at  $x = 0$ . Its right side is at  $x = 1$ . Its height is  $h(1)$  which turns out to be 8. Its width is the difference between the locations of the x coordinates of its left and right side –

that is  $1 - 0 = 1$ . Another way to say this would be,  $x_0 = 0$   
 $x_1 = 1, \Delta x = x_1 - x_0 = 1$ . Its area is height times width or  $8 * 1$   
 or  $h(1) * \Delta x$ .

**Label  $x_0$  and  $x_1$  on the graph. Label the height and width of the first rectangle ( $h(1) = 8$  and  $\Delta x = 1$ .)**

7. Refer back to 6 as you fill in the following sentences. Look at the second box. Its width is 1. Its left side is at \_\_\_\_\_ . Its right side is at \_\_\_\_\_ . It's height is \_\_\_\_\_ which turns out to be \_\_\_\_\_ . Its width is the difference between the locations of \_\_\_\_\_  
 \_\_\_\_\_ -- that is \_\_\_\_\_ . Another way to say this would be  $x_1 =$  \_\_\_\_\_ ,  $x_2 =$  \_\_\_\_\_ ,  $\Delta x =$  \_\_\_\_\_ . It's area is height times width or \_\_\_\_\_ or \_\_\_\_\_ .  
 Label  $x_2$  ,  $\Delta x$ , and  $h(x_2)$  on the second box.

8. You cannot see the third box because \_\_\_\_\_  
 \_\_\_\_\_ , and its area is \_\_\_\_\_ .

9. If I asked you to find  $\sum_{k=1}^3 (9 - k^2) \Delta x$  you would be finding the sum of the areas of these three rectangles.

Find  $\sum_{k=1}^3 (9 - k^2) \Delta x =$  \_\_\_\_\_

10. **On your picture, color the area you found in 9.**
11. The area estimate you came up with in 9 is *too high/too low* (circle one) compared to the actual area under the curve.
12. In order to get a better estimate of this area, we could make more boxes, each with a smaller width.

```
rightbox(h(x),x=0 .. 3,12)
```

You do not have to draw this picture. Just refer to it as you complete step 13.

13. Refer back to 6. as you fill in the following sentences. Look at the first box. Its width is \_\_\_\_\_. Its left side is at \_\_\_\_\_. Its right side is at \_\_\_\_\_. Its height is \_\_\_\_\_ which turns out to be \_\_\_\_\_. Its width is the difference between the locations of \_\_\_\_\_ -- that is \_\_\_\_\_. Another way to say this would be  $x_0 =$  \_\_\_\_\_,  $x_1 =$  \_\_\_\_\_,  $\Delta x =$  \_\_\_\_\_. Its area is height times width or \_\_\_\_\_ or \_\_\_\_\_.
- Label  $x_2$ ,  $\Delta x$ , and  $h(x_2)$  on the second box.

14. I could ask you to go through step 13 for all twelve boxes, add them all up to get an approximation for the area under the curve. But that would be too tedious. Let's get Maple to do the work.

```
delta_x := 1/4
approx_area := sum(h(k*delta_x)*delta_x, k = 1 ..12)
evalf(%)
```

Look at the result, and explain what each of the previous 4 command lines means.<sup>3</sup>

`delta_x := 1/4` means \_\_\_\_\_

`approx_area :=` means \_\_\_\_\_

`sum(h(k*delta_x)*delta_x, k = 1 ..12)` means \_\_\_\_\_

`evalf(%)` means \_\_\_\_\_

15.  $\sum_{k=1}^{12} (9 - (k * (\Delta x))^2) \Delta x$  is what the computer just did for you.

16. What number did Maple return as the answer, and what does it mean?

\_\_\_\_\_

\_\_\_\_\_

17. Which answer (the one from # 9, or from #16) is a more accurate estimation of the area under the curve? \_\_\_\_\_  
Why?

\_\_\_\_\_

\_\_\_\_\_

18. What could be done to make the area approximation even more accurate?

\_\_\_\_\_

\_\_\_\_\_

19. Let's change the approx\_area definition to make it easier to make more and more rectangles.

```
n:=20
delta_x := 3/n
approx_area := sum(h(k*delta_x)*delta_x, k = 1 ..n)
evalf(%)
```

20. Change the values of n (from 20 to: 100, 1000, 10000) to complete the following table to 5 decimal places:

<u>n</u>	<u>approx area</u>
20	_____
100	_____
1000	_____
10000	_____
<u>n -&gt; ∞</u>	_____

21. If everything was done correctly, it should be easy to fill in the last row by guessing.

